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B.Sc. (NCF-NEP) (Sem.-IV) Examination April - 2026

Major-3-Mathematics (Number Theory-I) (Paper - X)

[Total Marks: 50

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) Sketch neat and labelled diagram wherever necessary. (3) Figures to the right indicate full marks of the question. (4) All questions are compulsory. (5) Follow usual notations.	Student's Signature

Q.1. Answer the following questions (Any Five). **10**

1. Define : Greatest Common Divisor.
2. If $a|b$and $c|d$ then prove that $ac|bd$.
3. Find prime factorization of 17400.
4. Find gcd (119,272)
5. Check whether 403 is a prime composite number.
6. Prove that any prime number of the form $n^3 - 1$ is 7.
7. Find the reminder when 2^{50} is divided by 7.
8. If $N = 15x1724$ is divisible by 11 then find the value of x.

Q.2. Answer the following questions (Any two). **10**

1. If $a = bq + r$then gcd (a,b) = gcd (b,r)
2. Given integers a and b be any two integers not both of them zero, then gcd (a, b) =1 iff there exists integers x & y such that $ax + by = 1$
3. For any arbitrary integers "a" verify, if a is odd then prove that, $32|(a^2 + 3)(a^2 + 7)$

Q.3. Answer the following questions (Any two). **10**

1. Determine all the solution in the integers of the Diophantine equation : $2x + 5y = 1$
2. If $a_1\, a_2, \ldots\ldots\ldots\ldots, a_n$ are integers and p is prime numbers such that, $p|a_1.a_2\ldots\ldots\ldots\ldots a_n$ then $p|a_k$; for some k, where $1 \le k \le n$.
3. Show that the number $\sqrt{2}$ is an irrational number.

Q.4. Answer the following questions (Any two). **10**

1. If P_n. is the nth prime number, then show that $P_n < 2^{2^{n+1}}$
2. Prove that integer of the form $n^4 + 4$ is a composite number, $n > 1$.
3. Prove that there are infinitely many prime numbers.

Q.5. Answer the following questions (Any two). **10**

1. If $ca \equiv cb \ (mod\ n)$ then prove that, $a \equiv b \ (mod\ \frac{n}{d})$ where $d = \gcd$ (c,n)
2. If $a \equiv b \ (mod\ n)$ then prove that, gcd (a, n) = gcd (b, n)
3. What is the remainder of the sum $1^5 + 2^5 + 3^5 + \ldots\ldots\ldots\ldots + (1000)^5$ is divided by 4?